Making Mazes

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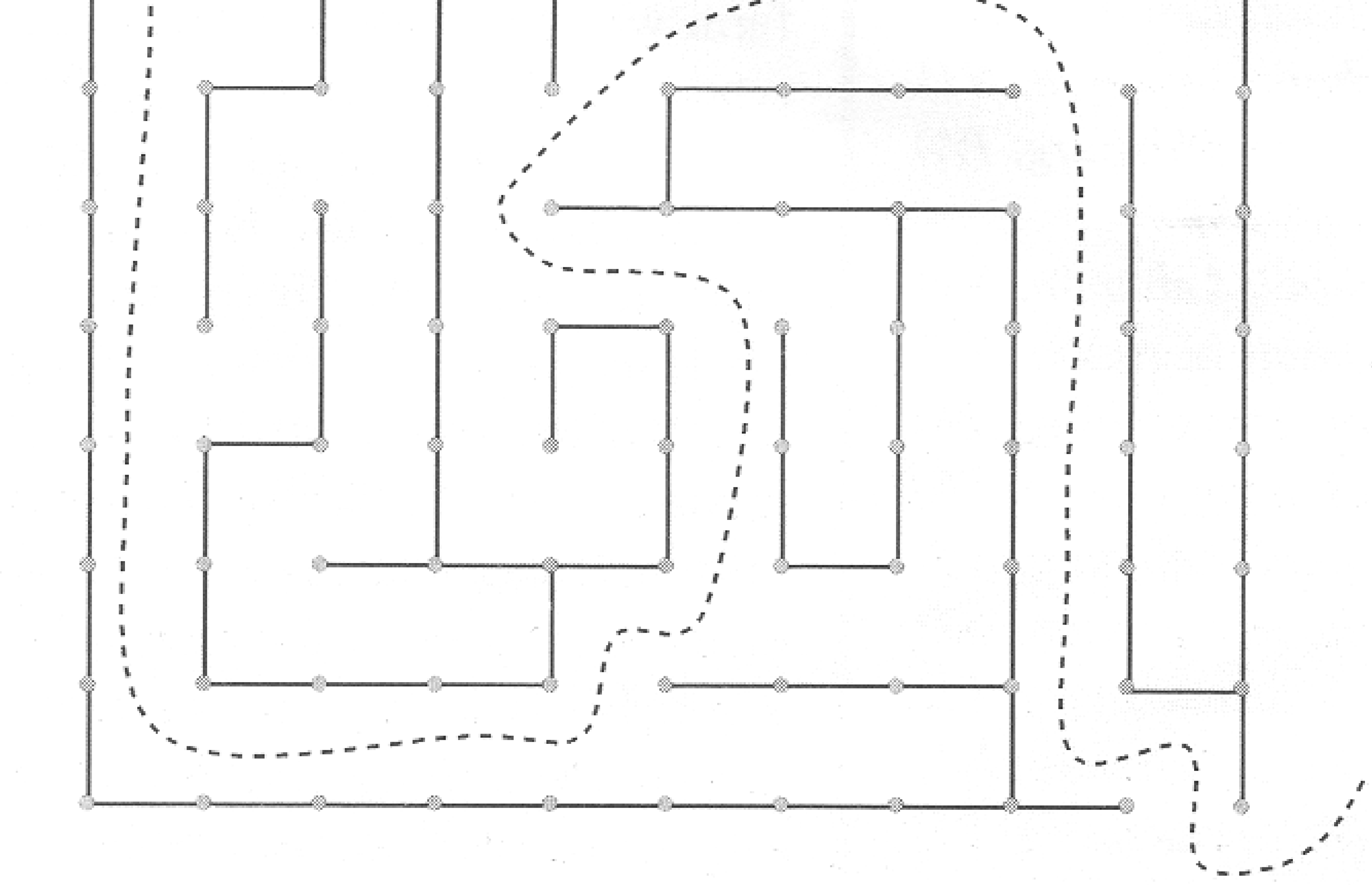
There is a lot of recreational mathematics involving mazes. Solving mazes is part of the fun, but drawing them can also be enjoyable, and poses some interesting questions. Let's look at mazes and how they work.

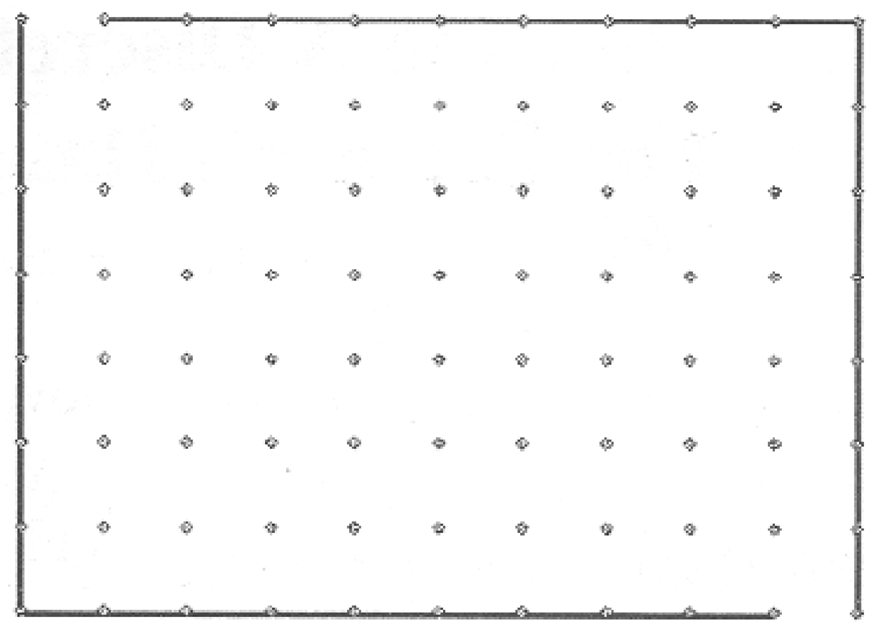
A maze has an entrance and an exit, and if it is a good maze, it will also have a way through, from the entrance to the exit! So if we imagine a person walking through a life-sized maze, trailing a red string (or a dotted line, as in the figure below) behind him from the entrance to the exit, we discover that a maze consists of two parts which don't touch one another at all; for the red string separates them. And each of these two parts is a tree, that is, a graph with no cycles.

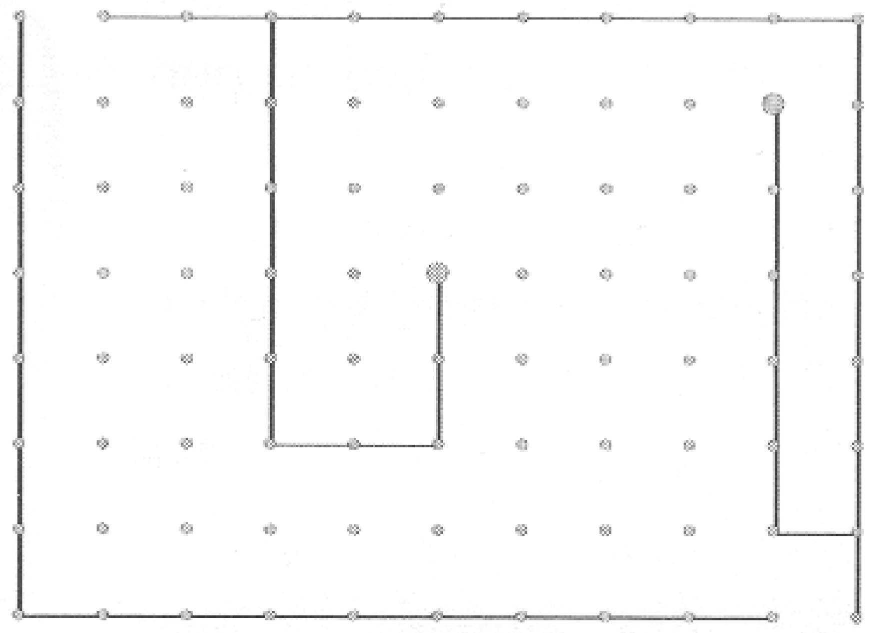
Thus, to build a maze, one simply draws two trees which don't touch one another, and specifies an entrance and an exit. And to solve the maze, one simply walks between the two parts. But to make the maze challenging, the parts should have lots of corridors to confuse the maze-solver.

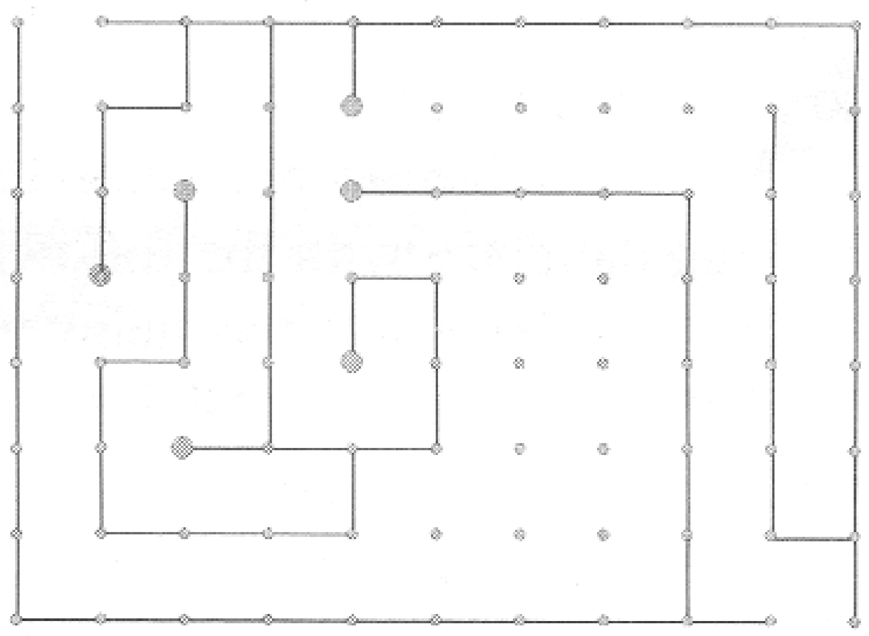
So, here's one way to build good mazes: Start with a rectangle, drawn on square dot paper, but with two "doors" cut into it (top figure). Then grow trees! A tree is grown by selecting an unused vertex within the rectangle (large dots in the middle figure) and walking horizontally and/or vertically from vertex to vertex, until a used vertex is reached. The middle figure shows two branches grown. Note that all the vertices on the two trees are now considered "used" vertices, and so other branches can grow onto them. (The bottom figure shows 6 branches so far; two of them are just single edges which grew onto another branch.) We continue growing branches, always starting at an unused vertex and walking to a used vertex, and then stopping there. When all the vertices inside the rectangle are used, we have a maze, as shown below.

Note that the entrance and exit can be anywhere on the rectangle, and you'll still get a maze. In fact, you can take a maze, close the exit, and erase a border edge to create a new exit somewhere else, and you get a new maze!









If the maze has a dots along the left side and b dots along the top, then the student will have to draw a total of $(a-2)(b-2)$ inside edges in a complete maze. Thus the maze on the left has $(8-2)(11-2) = 54$ edges inside the rectangle.

One classic technique for solving a maze is to imagine walking through the maze, always keeping your right hand on the wall. This method will always work on the mazes described here. (Try it out on the maze to the left!) Sometimes it will lead you to a dead end, but it will also lead you out of that dead end, never to return.

But what about blindfolded? Suppose your students have drawn a maze on the blackboard, placed a marker somewhere in the maze to mark your position, and then challenged you to get out of the maze. (We assume that they block off the entrance so you don't accidentally walk out that way!) All you can do is call directions (north, south, east or west) and the class responds "no" if a wall is blocking you in that direction and "yes" if there is an opening in that direction, in which case they move the marker. How do you get out? Tammy Wopnford was kind enough to let me try this with her 5th graders at Sonoran Sky Elementary School in Scottsdale, and we had a lot of fun.

Here is a nice algorithm (look at the compass below as you follow this algorithm): Say "North" first. If the students say "yes," then turn clockwise around the compass to "East," and make that your next guess. But if they say "No" to your "North," then turn counter-clockwise and make "West" your next guess. In general, whenever they say "Yes," turn clockwise for your next guess, and whenever they say "No," turn counter-clockwise for your next guess. This will get you out of the maze, guaranteed, just as if you were keeping your right hand on the wall. And the kids will never guess your trick!